

**HIGHER SECONDARY
FIRST TERMINAL SECOND YEAR EXAMINATION 2018-19
MATHEMATICS (SCIENCE)**

Maximum : 80 Scores
Time 2 ½ hours
Cool off time : 15 minutes

HSE II

Answer any six from questions 1 to 7. Each question carries 3 score.

1. Construct a 3×2 matrix $A = [a_{ij}]$ whose elements are given by $a_{ij} = \frac{(i+j)^2}{2}$ (3)

$$a_{ij} = \frac{(i+j)^2}{2}$$

$$a_{11} = \frac{(1+1)^2}{2} = 2; a_{12} = \frac{(1+2)^2}{2} = 4.5$$

$$a_{21} = \frac{(2+1)^2}{2} = 4.5; a_{22} = \frac{(2+2)^2}{2} = 8$$

$$a_{31} = \frac{(3+1)^2}{2} = 8; a_{32} = \frac{(3+2)^2}{2} = 12.5$$

$$\therefore A = \begin{bmatrix} 2 & 4.5 \\ 4.5 & 8 \\ 8 & 12.5 \end{bmatrix}$$

2. Show that the relation R on Z defined by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation. (3)

i. If $|a - a| = 0$, is even $\forall a \in R \Rightarrow R$ is reflexive.

ii. If $|a - b|$ is even $\Rightarrow |b - a| = |-(a - b)| = |a - b|$ is even $\forall a, b \in R \Rightarrow R$ is symmetric.

iii. If $|a - b|$ is even and if $|b - c|$ is even \Rightarrow

$$|(a - b) + (b - c)| = |a - c| \text{ is also even } \forall a, b, c \in R \Rightarrow R \text{ is transitive.}$$

Hence R is an equivalence relation.

3. If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -3 & 0 \\ 5 & -2 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & -1 \\ -6 & 2 \\ 7 & 4 \end{bmatrix}$

a) $AB = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -3 & 0 \\ 5 & -2 & 1 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ -6 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 5 + -18 + 14 & -1 + 6 + 8 \\ 10 + 18 + 0 & -2 - 6 + 0 \\ 25 + 12 + 7 & -5 - 4 + 4 \end{bmatrix} = \begin{bmatrix} 1 & 13 \\ 28 & -8 \\ 44 & -5 \end{bmatrix}$ (2)

b) Here multiplying all the elements of 2nd row by 2, hence in the product matrix, multiplying all elements

by 2, we have $AB = \begin{bmatrix} 1 & 13 \\ 56 & -16 \\ 44 & -5 \end{bmatrix}$ (1)

4. a) b) $|\sin x|$ (1)

b) Since modulus function and trigonometric functions are continuous everywhere, then the composite function $|\sin x|$ is continuous everywhere. (1)

c) It is clear from the graph that $|\sin x|$ is not differentiable at $x = n\pi$, where n is an integer. (1)

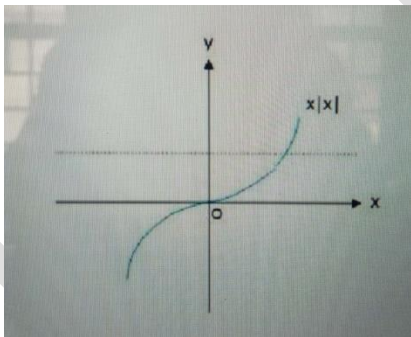
5. Consider the matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

a) $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} = 2(0-20) - 3(-42-4) + 5(30-0) = -40 - 138 + 150 = -28$ (1)

b) $|adjA| = |A|^{n-1} = (-28)^{3-1} = (-28)^2 = 784$ (1)

c) $|2A| = 2^3 |A| = 8(-28) = -224$ (1)

6. a)



b) Yes. In the graph, the line parallel to the x axis intersects the graph only one point. (1)

c) No. Every y element does not have pre-image. (1)

7. $f(x) = x^2 + 2, g(x) = 2x + 3$

$f \circ g = f[g(x)] = f(2x+3) = (2x+3)^2 + 2 = 4x^2 + 12x + 9 + 2 = 4x^2 + 12x + 11$

$g \circ f = g[f(x)] = g(x^2 + 2) = 2(x^2 + 2) + 3 = 2x^2 + 4 + 3 = 2x^2 + 7$ (3)

Answer any eight from questions 8 to 17. Each question carries 4 score.

8. Let $f(x) = 4x^2 + 12x + 15$

Let y be the arbitrary element of range f . Then $y = 4x^2 + 12x + 15$

$y = 4x^2 + 12x + 15$, for some $x \in N$.

Now $y = 4x^2 + 12x + 9 + 6 = (2x+3)^2 + 6 \Rightarrow y - 6 = (2x+3)^2$

$\Rightarrow 2x+3 = \sqrt{y-6} \Rightarrow 2x = \sqrt{y-6} - 3 \Rightarrow x = \frac{\sqrt{y-6} - 3}{2}$, as $y \geq 6$

Let us define $g : S \rightarrow N$ by $g(y) = \frac{\sqrt{y-6} - 3}{2}$

Now $g \circ f(x) = g[f(x)] = g((2x+3)^2 + 6) = \frac{\sqrt{(2x+3)^2 + 6 - 6} - 3}{2} = \frac{2x+3-3}{2} = x$

and

$f \circ g(y) = f[g(y)] = f\left(\frac{\sqrt{y-6} - 3}{2}\right) = \left(2 \times \frac{\sqrt{y-6} - 3}{2} + 3\right)^2 + 6 = (\sqrt{y-6})^2 + 6 = y - 6 + 6 = y$

$g \circ f = I_N$ and $f \circ g = I_S$. This implies that f is invertible with $f^{-1} = g$ (4)

9. $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & -1 \end{bmatrix}; B = \begin{bmatrix} 1 & -2 \\ 3 & 2 \\ 2 & -1 \end{bmatrix}$

a) $AB = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 3 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1+6+6 & -2+4-3 \\ 3+6-2 & -6+4+1 \end{bmatrix} = \begin{bmatrix} 13 & -1 \\ 7 & -1 \end{bmatrix}$ (1)

b) $B' = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 2 & -1 \end{bmatrix}; A' = \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & -1 \end{bmatrix}$ (1)

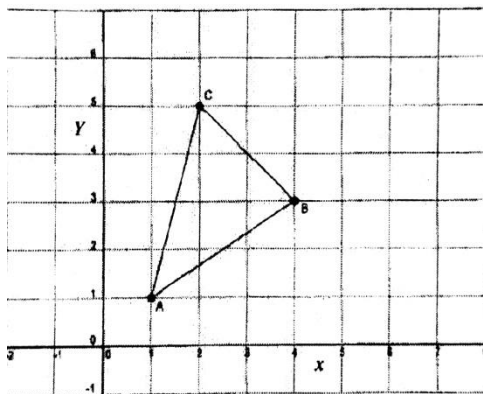
c) $(AB)' = \begin{bmatrix} 13 & 7 \\ -1 & -1 \end{bmatrix}$(1)

$B' = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1+6+6 & 3+6-2 \\ -2+4+-3 & -6+4+1 \end{bmatrix} = \begin{bmatrix} 13 & 7 \\ -1 & -1 \end{bmatrix}$(2)

From (1) and (2), $(AB)' = B'A'$. Hence proved (2)

10. a) Find the area of the triangle ABC shown in figure using determinants

(2)



The points are : A(1,1) ; B (4,3) ; C (2,5)

$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \begin{vmatrix} 1 & 4 & 2 \\ 1 & 3 & 5 \\ 1 & 1 & 1 \end{vmatrix} \\ &= \frac{1}{2} [1(3-5) - 4(1-5) + 2(1-3)] = \frac{1}{2} [-2 + 20 - 4] = \frac{14}{2} = 7 \text{ sq.units} \end{aligned}$$

b) Area of $\triangle ABD = 7$ sq cm

$$\frac{1}{2} \begin{vmatrix} 1 & 4 & k \\ 1 & 3 & 6 \\ 1 & 1 & 1 \end{vmatrix} = 7$$

$$1(3-6) - 4(1-6) + k(1-3) = 14$$

$$-3 + 20 - 2k = 14 \Rightarrow 17 - 2k = 14$$

$$17 - 14 = 2k \Rightarrow 3 = 2k \Rightarrow k = \frac{3}{2}$$

(2)

$$11. A = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 2 & 3 \\ 2 & 3 & -2 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 6 & 0 & 3 \\ 0 & 4 & 6 \\ 3 & 6 & -4 \end{bmatrix}$$

$$P' = \frac{1}{2} \begin{bmatrix} 6 & 0 & 3 \\ 0 & 4 & 6 \\ 3 & 6 & -4 \end{bmatrix} = P$$

∴ P is symmetric

$$\text{Now, } Q = \frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} = Q$$

Q is skew symmetric.

$$\begin{aligned} \text{Now } P + Q &= \frac{1}{2} \begin{bmatrix} 6 & 0 & 3 \\ 0 & 4 & 6 \\ 3 & 6 & -4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 6 & -2 & 4 \\ 2 & 4 & 6 \\ 2 & 6 & -4 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & -2 \end{bmatrix} = A, \text{ the square matrix.} \end{aligned}$$

(4)

12. Let $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{k \times \cos x}{\pi - 2x} \right) = \frac{k \cos \frac{\pi}{2}}{\pi - 2 \frac{\pi}{2}} = \frac{0}{0} \text{ (indeterminate form)}$$

Put $x = \frac{\pi}{2} + h$

As $x \rightarrow \frac{\pi}{2}; h \rightarrow 0$

$$\lim_{h \rightarrow 0} \left[\frac{k \cos \left(\frac{\pi}{2} + h \right)}{\pi - 2 \left(\frac{\pi}{2} + h \right)} \right] = \lim_{h \rightarrow 0} \frac{k(-\sin h)}{\pi - \pi - 2h} = \lim_{h \rightarrow 0} \frac{-k \sin h}{-2h} = \frac{k}{2} \times 1 = \frac{k}{2}$$

Now, $f\left(\frac{\pi}{2}\right) = 3$

f is continuous at $\frac{\pi}{2}$

$$\lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) \Rightarrow \frac{k}{2} = 3 \Rightarrow k = 6$$

(4)

13. a) **The question is wrong.** Correct question is : The value of $\tan^{-1} \tan\left(\frac{3\pi}{4}\right) = \dots\dots\dots$

(1)

$$\begin{aligned} \tan^{-1} \tan\left(\frac{3\pi}{4}\right) &= \tan^{-1} \tan\left(\pi - \frac{\pi}{4}\right) = \tan^{-1} \tan\left(\pi - \frac{\pi}{4}\right) \\ &= \tan^{-1} \left[-\tan\left(\frac{\pi}{4}\right) \right] = \tan^{-1} \tan\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4} \end{aligned}$$

$$\begin{aligned}
 \text{b) LHS} &= 2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left[\frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2}\right] + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{1}{1-\frac{1}{4}}\right) + \tan^{-1}\left(\frac{1}{7}\right) \\
 &= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left[\frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \times \frac{1}{7}}\right] = \tan^{-1}\left[\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right] = \tan^{-1}\left[\frac{31}{17}\right] \\
 &= \tan^{-1}\left(\frac{31}{17}\right) = \text{RHS}
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 \text{14. a) Let } A &= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \\
 A^2 &= \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \\
 \text{LHS} = A^2 - 4A + I &= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = \text{RHS}
 \end{aligned} \tag{2}$$

$$\text{b) } A^2 - 4A + I = 0 \Rightarrow A - 4I + A^{-1} = 0 \quad \parallel \text{ xing by } A^{-1}$$

$$\text{Hence, } A^{-1} = 4 - A = 4I - A = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} \tag{2}$$

$$\text{15. a) } \begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$$

$$x^2 - 36 = 36 - 36 \Rightarrow x^2 = 36 \Rightarrow x = \pm 6$$

Ans: A

$$\text{b) LHS} = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

$$R_1 \rightarrow R_1; R_2 \rightarrow R_2 - R_1; R_3 \rightarrow R_3 - R_1$$

$$\begin{aligned}
 LHS &= \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix} \\
 &= 1 \times (b-a) \times (c-a) \begin{vmatrix} 1 & -c \\ 1 & -b \end{vmatrix} \\
 &= (b-a)(c-a)(-b--c) \\
 &= (b-a)(c-a)(c-b) = (a-b)(b-c)(c-a) = RHS
 \end{aligned}
 \tag{3}$$

$$16. a) \quad 2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots\dots\dots(1)$$

$$3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \dots\dots\dots(2)$$

$$(1) \times 3 - (2) \times 2 \Rightarrow$$

$$6X + 9Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$$

$$6X + 4Y = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$$

$$5Y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$$

$$Y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

$$\text{in (1)} \Rightarrow 2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3Y$$

$$X = \frac{1}{2} \left(\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3 \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix} \right)$$

$$= \frac{1}{2} \left(\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & 3 \end{bmatrix}$$

(4)

17. $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Let $A = IA$

$$\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$\begin{bmatrix} 1 & 2 \\ 0 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

$$R_2 \rightarrow \frac{-1}{5} R_2$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{-1}{5} \end{bmatrix} A$$

$$A^{-1} = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

(4)

Answer any eight from questions 18 to 24. Each question carries 6 score.

18.

$$AX = B$$

$$\text{Where } A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} = 1(1+3) - 1(2-3) + 1(2-1)$$

$$= 1(4) + 1(5) + 1(1)$$

$$= 4 + 5 + 1 = 10$$

$$A_{11} = 4 \quad A_{12} = -5 \quad A_{13} = 1$$

$$A_{21} = +2 \quad A_{22} = 0 \quad A_{23} = -2$$

$$A_{31} = 2 \quad A_{32} = +5 \quad A_{33} = 3$$

$$\text{adj}A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$\therefore x = 2; y = -1; z = 1$$

(6)

19. a) $\sin(\sin^{-1}x + \cos^{-1}x) = \sin \frac{\pi}{2} = 1$

(1)

b) $\sin^{-1}\left(\sin \frac{3\pi}{5}\right) = \sin^{-1}\left(\sin\left(\pi - \frac{2\pi}{5}\right)\right) = \sin^{-1}\left(\sin \frac{2\pi}{5}\right) = \frac{2\pi}{5} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(2)

c) $\sin(\tan^{-1}x) = \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right) = \frac{x}{\sqrt{1+x^2}}$

(1)

d) $LHS = 2 \sin^{-1}\left(\frac{3}{5}\right) = 2 \tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1 - \left(\frac{3}{4}\right)^2}\right) = \tan^{-1}\left(\frac{\frac{6}{4}}{1 - \frac{9}{16}}\right)$

$$= \tan^{-1} \left(\frac{\frac{6}{4}}{\frac{16-9}{16}} \right) = \tan^{-1} \left(\frac{6}{4} \times \frac{16}{7} \right) = \tan^{-1} \left(\frac{24}{7} \right) = RHS \quad (2)$$

20. a) $\cos^{-1} \left(\frac{\sqrt{3}}{2} \right) = \frac{\pi}{6} \Rightarrow \cos^{-1} \left(-\frac{\sqrt{3}}{2} \right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \in [0, \pi]$ (1)

b) $y = \cos^{-1} \left(\frac{2x}{1+x^2} \right) = \cos^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) \quad \parallel \text{ put } x = \tan \theta \Rightarrow \theta = \tan^{-1} x$

$$\cos^{-1} \sin 2\theta = \cos^{-1} \cos \left(\frac{\pi}{2} - 2\theta \right) = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2 \tan^{-1} x \quad (3)$$

c) $y = \cos^{-1} \left(\frac{2x}{1+x^2} \right) = \frac{\pi}{2} - 2 \tan^{-1} x$

$$\frac{dy}{dx} = 0 - 2 \times \frac{1}{1+x^2} = -\frac{2}{1+x^2} \quad (2)$$

21.

a) $y = \cos(x^3), \sin^2(x^5)$ (2)

$$\frac{dy}{dx} = \cos(x^3) \cdot \cos(x^5) \cdot 5x^4 + \sin(x^5) \cdot -\sin(x^3) \cdot 3x^2$$

$$= 5x^4(x^5)\cos(x^3) - 3x^2 \sin(x^5) \sin(x^3)$$

b) $x^2 + y^2 = 100$ (2)

diff wrt x

$$2x + 2y \frac{dy}{dx} = 0 \Rightarrow 2y \frac{dy}{dx} = -2x$$

$$\therefore \frac{dy}{dx} = \frac{-2}{2y} = -\frac{x}{y}$$

c) $y = (\log x)^x + x^{\log x}$ (2)

put $u = (\log x)^x; v = x^{\log x}$

Then $y = u + v$

And $\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} \dots \dots \dots (1)$

Let $u = (\log x)^x$

$$\log u = x \log(\log x)$$

$$\frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log(\log x).$$

$$\therefore \frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right]$$

Now, $V = x \log^x$

$$\log V = \log x^{\log x} = \log x \cdot \log x = (\log x)^2$$

$$\frac{1}{V} \cdot \frac{dV}{dx} = V \cdot \frac{2 \log x}{x} = x^{\log x} \cdot \frac{2 \log x}{x}$$

$$= x^{\log x - 1} \cdot 2 \log x$$

(1) \Rightarrow

$$\frac{dy}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + x^{\log x - 1} \cdot 2 \log x$$

22. a) $a * b = \frac{ab}{2}$

$$b * a = \frac{ba}{2} = \frac{ab}{2}$$

$\therefore *$ is commutative

$$\text{Again } a * (b * c) = a * \left(\frac{bc}{2} \right) = \frac{a \cdot \frac{bc}{2}}{2} = \frac{abc}{4}$$

$$(a * b) * c = \left(\frac{ab}{2} \right) * c = \frac{\frac{ab}{2} \cdot c}{2} = \frac{abc}{4}$$

$$\Rightarrow a * (b * c) = (a * b) * c$$

$\therefore *$ is associative. (2)

b) Consider the set $A = \{1, 2, 3, 4, 5\}$

i.

*	1	2	3	4	5
1	1	2	3	4	5
2	1	2	3	4	5
3	3	3	3	3	3
4	1	2	3	4	5
5	1	2	3	4	5

(2)

ii. The table has 25 cells, Fill one row and column of 3 as required 16 cells remain. Here repetition is allowed, so there are 5 raised to 16 (5^{16}) binary operations are there. (2)

23. a)

$$LHS = \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\frac{12}{13}$$

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$$

$$\therefore LHS = \cos^{-1}\left[\frac{4}{5} \times \frac{12}{13} - \sqrt{1-\left(\frac{4}{5}\right)^2} \sqrt{1-\left(\frac{12}{13}\right)^2}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \sqrt{1-\frac{16}{25}} \sqrt{1-\frac{144}{169}}\right] = \cos^{-1}\left[\frac{48}{65} - \sqrt{\frac{9}{25}} \sqrt{\frac{25}{169}}\right]$$

$$= \cos^{-1}\left[\frac{48}{65} - \frac{3}{5} \times \frac{5}{13}\right] = \cos^{-1}\left(\frac{48-15}{65}\right)$$

$$= \cos^{-1}\left(\frac{33}{65}\right) = RHS$$

(3)

b)

$$\tan^{-1}\left(\frac{2x+3x}{1-2x.3x}\right) = \frac{\pi}{4}$$

$$\frac{5x}{1-6x^2} = \tan \frac{\pi}{4} = 1$$

$$5x = 1 - 6x^2 \Rightarrow 6x^2 + 5x - 1 = 0$$

$$6x^2 + 6x - x - 1 = 0$$

$$6x(x+1) - 1(x+1) = 0$$

$$(x+1)(6x-1) = 0$$

$$x = -1 \text{ (or) } 6x = 1 \Rightarrow x = \frac{1}{6}$$

But $x = -1$ is impossible.

$$\therefore x = \frac{1}{6}$$

(3)

24. a) Find $\frac{dy}{dx}$ if $x = a\left(\cos t + \log \tan\left(\frac{1}{2}\right)\right)$, $y = a \sin t$

(3)

The question has a type mismatch. The correct question is:

Find $\frac{dy}{dx}$ if $x = a\left(\cos t + \log \tan\left(\frac{t}{2}\right)\right)$, $y = a \sin t$

$$x = a\left(\cos t + \log \tan \frac{t}{2}\right); y = a \sin t$$

$$\frac{dx}{dt} = a \left(-\sin t + \frac{1}{\tan \frac{t}{2}} \cdot \sec^2 \frac{t}{2} \cdot \frac{1}{2} \right) = a \left(\sin t + \frac{1}{\frac{\sin \frac{t}{2}}{\cos \frac{t}{2}}} \times \frac{1}{\cos^2 \frac{t}{2}} \times \frac{1}{2} \right)$$

$$= a \left(-\sin t + \frac{1}{2 \sin \frac{t}{2} \cos \frac{t}{2}} \right) = a \left(-\sin t + \frac{1}{\sin t} \right)$$

$$= a \left(\frac{-\sin^2 t + 1}{\sin t} \right) = a \frac{\cos^2 t}{\sin t} = \frac{a \cos^2 t}{\sin t}$$

$$y = a \sin t$$

$$\frac{dy}{dx} = a \cos t$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = a \cos t \times \frac{\sin t}{a \cos^2 t} = \tan t$$

b) $y = \sin^{-1} x$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2} \cdot \frac{dy}{dx} = 1$$

diff wrt x

$$\sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{-x}{\sqrt{1-x^2}} = 0$$

$$(1-x)^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0 \quad \parallel \times \text{ing by } \sqrt{1-x^2}$$

Hence proved.

(3)



**Confidence and Hardwork
is the Best Medicine to Kill
the Disease called Failure .
It will Make You
Successful Person.....**