FIRST YEAR HIGHER SECONDARY EXAMINATION MARCH 2019

PART III

MATHEMATICS(SCIENCE)

Answer key

1. $A = \{2,3,5,7\}$

   $B = \{2,3,4,5,6,7,8\}$

   a) $C = \{2,3,5,7\}$

   b) 4

   c) Required probability $= \frac{4}{16} = \frac{1}{4}$

2. a)

$$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$
$$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$$

Subtracting we have,

$$(a+b)^4 - (a-b)^4 = 2\times 4a^3b + 2\times 4ab^3$$
$$= 8ab\left(a^2 + b^2\right)$$

b) $\therefore \left(\sqrt{3}+\sqrt{2}\right)^4 - \left(\sqrt{3}-\sqrt{2}\right)^4 = 8\sqrt{3}\sqrt{2}\left[\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2\right] = 8\sqrt{6}\,(3+2) = 40\sqrt{6}$

3. Let $\sqrt{3+4i} = x + iy$ ....(1)

Squaring,

$$3 + 4i = (x+iy)^2$$

Equating real and imaginary parts,

$$x^2 - y^2 = 3 \text{.....................}(2)$$
$$2xy = 4 \text{....................}(3)$$
$$w.k.t, \left(x^2+y^2\right)^2 = \left(x^2-y^2\right)^2 + (2xy)^2$$
$$= 9 + 16 = 25$$
$$x^2 + y^2 = 5 \text{....................}(4)$$
$$(2)+(4) \Rightarrow$$
$$x^2 - y^2 = 3$$
$$x^2 + y^2 = 5$$
$$2x^2 = 8$$

$x^2 = 4$

$x = \sqrt{4} = \pm 2$

In (4) , $4 + y^2 = 5 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$

$\therefore \sqrt{3+4i} = \pm(2+i)$

4. Let the terms be $\frac{a}{r}, a, ar$

   $\frac{a}{r} + a + ar = \frac{13}{12}$ …………..(1)

   $\frac{a}{r} \times a \times ar = -1 \Rightarrow a^3 = -1 \Rightarrow a = -1(real)$

   in (1), we have,

   $\frac{-1}{r} + (-1) + (-1)r = \frac{13}{12} \Rightarrow \frac{1}{r} + 1 + r = -\frac{13}{12}$

   $12 + 12r + 12r^2 = -13r \Rightarrow 12r^2 + 25r + 12 = 0$

   $4r(3r+4) + 3(3r+4) = 0 \Rightarrow (3r+4)(4r+3) = 0$

   $3r + 4 = 0 \Rightarrow r = -\frac{4}{3}$ *or* $4r + 3 = 0 \Rightarrow r = -\frac{3}{4}$

   Hence the three terms of the G.P are: $\frac{3}{4}, -1, \frac{4}{3}$ *when* $r = -\frac{4}{3}$ and $\frac{4}{3}, -1, \frac{3}{4}$ *when* $r = -\frac{3}{4}$.

5. $\sin 5x + \sin x + \sin 3x = 0 \Rightarrow 2\sin 3x \cos 2x + \sin 3x = 0$

   $\sin 3x(2\cos 2x + 1) = 0$

   If $\sin 3x = 0 \Rightarrow 3x = n\pi \Rightarrow x = \frac{n\pi}{3}, n \in Z$

   If $2\cos 2x + 1 = 0 \Rightarrow \cos 2x = -\frac{1}{2} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\frac{2\pi}{3}$

   $\therefore x = 2n\pi \pm \frac{2\pi}{3}, n \in Z$

6. a) $f(x) = \sin x$

   b) $f(x) = \sin x,\ f\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1$

$$\lim_{x\to\frac{\pi}{2}} \frac{f(x)-f\left(\frac{\pi}{2}\right)}{x-\frac{\pi}{2}} = \lim_{x\to\frac{\pi}{2}}\left(\frac{\sin x-1}{x-\frac{\pi}{2}}\right) = \left(\frac{0}{0}\right)$$

$put\ x = \frac{\pi}{2}+h$

$As\ x \to \frac{\pi}{2}, h \to 0$

$$The\ \text{expn} = \lim_{h\to 0}\left(\frac{\sin\left(\frac{\pi}{2}+h\right)-1}{\frac{\pi}{2}+h-\frac{\pi}{2}}\right) = \lim_{h\to 0}\left(\frac{\cos h-1}{h}\right)$$

$$= -\lim_{h\to 0}\left(\frac{1-\cos h}{h}\right) = -1\times\lim_{h\to 0}\left(\frac{1-\cos h}{h}\right) = -1\times 0 = 0$$

7. a) General term $t_{r+1} = {}^5C_r\ \left(x^2\right)^{5-r}\left(\frac{1}{x}\right)^r = {}^5C_r\ x^{10-2r}.x^{-r} = {}^5C_r \times x^{10-3r}$

   b) $t_{r+1} = {}^nC_r\ \left(x^2\right)^{n-r}\left(\frac{1}{x}\right)^r = {}^nC_r\ x^{2n-2r}.x^{-r} = {}^nC_r \times x^{2n-3r}$

   To find the term independent of x, $2n-3r=0 \Rightarrow 2n=3r \Rightarrow r=\frac{2n}{3} \Rightarrow n=18$

8. Let A = set of students who were listed as taking apple juice.
   Let B = set of students who were listed as taking orange juice.
   a) $n(A\cup B) = n(A)+n(B)-n(A\cap B) = 100+150-75 = 175$
   b) $n(A-B) = n(A)-n(A\cap B) = 100-75 = 25$
   c) $n(A'\cap B') = n(U)-n(A\cup B) = 400-175 = 225$

9. The function is $f(x) = \begin{cases} 1, if\ \ x \ge 0 \\ x, if\ \ x < 0 \end{cases}$
   a) Domain $= R$
      Range $(-\infty, 1]$
   b) $f(0)=1$ and $f(-0.01) = -0.01$
   c) LHL = $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x = 0$
      LHL = $RHL = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(1) = 1$
   d) Since $LHL \ne RHL$, $f(x)$ does not exist at $x = 0$.

10. a) $A=\{-1,1\}$

$A\times A=\{(-1,-1),(-1,1),(1,-1),(1,1)\}$

b) No. of relations from A to A $=2^4=16$

c) The functions whose range $\{-1,1\}$ are: $f_1=\{(-1,-1),(1,1)\}$; $f_2=\{(-1,1),(1,-1)\}$

11. a) *Let* $P(n): n(n+1)(n+5)$ *is a multiple of* 3

$P(1): 1(1+1)(1+5)=12$, *is a multiple of* 3

$\therefore P(1)$ *is true.*

Assume that $P(k)$ *be true.*

$P(k): k(k+1)(k+5)$ *is a multiple of* 3

$\Rightarrow k(k+1)(k+5)=3M$ .... (1)

To prove that $P(k+1)$ *is true.*

$P(k+1): (k+1)(k+2)(k+6)$ *is a multiple of* 3

$\Rightarrow (k+1)(k+2)(k+5+1)=(k+1)(k+2)(k+5)+(k+1)(k+2)1$

$=k(k+1)(k+5)+2(k+1)(k+5)+(k+1)(k+2)$

$=3M+(k+1)\left[2(k+5)+k+2\right]=3M+(k+1)[3k+12]$

$=3M+3(k+1)[k+4]=3\left[M+(k+1)(k+4)\right]$, is divisible by 3.

$\therefore P(k+1)$ *is true.*

Hence, $P(n)$ *is true for all* $n\in N$.

12. $|z|=2$ and $\arg(z)=\dfrac{4\pi}{3}$

a) $z=r(\cos\theta+i\sin\theta)=2\left(\cos\dfrac{4\pi}{3}+i\sin\dfrac{4\pi}{3}\right)$

$=2\left(-\cos\dfrac{\pi}{3}-i\sin\dfrac{\pi}{3}\right)=2\left(-\dfrac{1}{2}-i\dfrac{\sqrt{3}}{2}\right)=-1-i\sqrt{3}$

b) $\bar{z}=-1+i\sqrt{3}$

c) $\bar{z}^2=\left(-1+i\sqrt{3}\right)^2=1-i2\sqrt{3}+i^2(3)=-2-i2\sqrt{3}=2\left(-1-i\sqrt{3}\right)=2z$

13. a) Total number of ways = ${}^{52}C_7$

b) P(all kings) $=\dfrac{{}^{4}C_4\times{}^{48}C_3}{{}^{52}C_7}=\dfrac{1}{7735}$

c) P(not a king card) $=1-P(\textit{all king cards})=1-\dfrac{1}{7735}=\dfrac{7734}{7735}$

14. a) If a number is not divisible by 3, it is not divisible by 9.

b) Let us assume that $\sqrt{7}$ is a rational number.

$\therefore\ \sqrt{7}=\frac{a}{b}$, where a and b are co-prime. i.e., a and b have no common factors, which implies that

$7b^2=a^2 \Rightarrow 7$ divides a.

$\therefore$ there exists an integer 'k' such that $a=7k$

$\therefore\ a^2=7^2k^2 \Rightarrow 7b^2=7^2k^2 \Rightarrow b^2=7k^2 \Rightarrow 7$ divides b.

i.e., 7 divides both a and b, which is contradiction to our assumption that a and b have no common factor. $\therefore$ our supposition is wrong. $\therefore\ \sqrt{7}$ is an irrational number.

15.

Class	Mid value x_i	fi	cf	$\|xi-m\|$	$fi\|xi-m\|$
0 – 10	5	6	6	23	138
10 – 20	15	7	13	13	91
20 – 30	25	15	28	3	45
30 – 40	35	16	44	7	112
40 – 50	45	4	48	17	68
50 – 60	55	2	50	27	54

N = 50 $\quad\quad \sum fi|xi-m|=508$

Median = size of $\left(\frac{N}{2}\right)^{th}$ observation

$=$ size of 25$^{\text{th}}$ observation

It is included in the cf =28.

$$\text{Median } = l+\frac{\frac{N}{2}-cf}{f}\times h$$

$$=20+\frac{25-13}{15}\times 10=28$$

$$MD(M)=\frac{\sum fi|xi-M|}{N}$$

$$=\frac{508}{50}=10.2$$

16. a) The word ASSASSINATION has

A	– 3
S	– 4
I	– 2
N	– 2
T	– 1
O	– 1

Total –13

Number of arrangements $= \frac{13!}{3!\times 4!\times 2!\times 2!} = 10810800$

b) If the vowels come together,

AAAIIO	– 1
S	– 4
M	– 2
T	– 1

Total – 8

Number of permutations $= \frac{8!}{4!\times 2!}\times\frac{6!}{3!\times 2!} = 840\times 60 = 50400$

17. a) $AB = \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}$

$BC = \sqrt{(-3)^2+3^2+0^2}$

$= \sqrt{1+1+16} = \sqrt{18}$

$AC = \sqrt{(-4)^2+2^2+(-4)^2}$

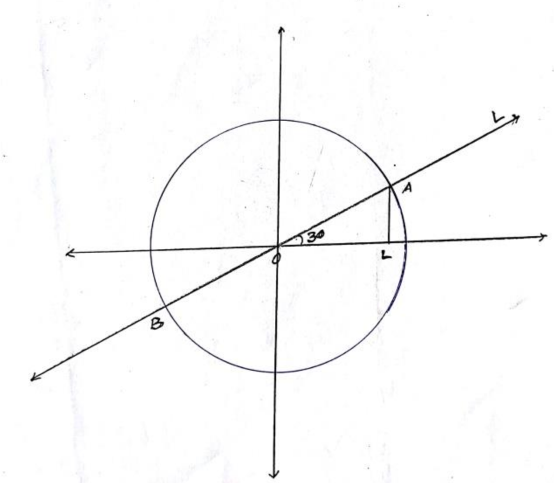
$= \sqrt{16+4+16}$

$= \sqrt{36} = 6$

$AB^2 + AC^2 = \left(\sqrt{18}\right)^2 + \left(\sqrt{18}\right)^2 = 18+18 = 36 = AC^2$

$\therefore ABC$ is a right triangle.

b) The question has a typographical error or out of syllabus.



18. a) slope of $L = \tan 30 = \frac{1}{\sqrt{3}}$

b) $AL \perp OX$

$\frac{OL}{OA} = \cos 30$

$OL = 1\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

$$\frac{AL}{OA} = \sin 30$$

$$AL = OA\sin 30 = (1)\left(\frac{1}{2}\right) = \frac{1}{2}$$

$$\therefore A \text{ is } \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$B \text{ is } \left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

Slope of the tangent at $A = \frac{-1}{\frac{1}{\sqrt{3}}} = -\sqrt{3}$

Equation of the tangent at A is $y - y_1 = m(x - x_1)$

$$y - \frac{1}{2} = -\sqrt{3}\left(x - \frac{\sqrt{3}}{2}\right)$$

$$y - \frac{1}{2} = -\sqrt{3}x + \frac{3}{2}$$

$$2y - 1 = -2\sqrt{3x} + 3$$

$$2y - 1 + 2\sqrt{3}x - 3 = 0$$

$$2\sqrt{3}x + 2y - 4 = 0$$

$$\sqrt{3}x + y - 2 = 0$$

19. a) Slope of $L_1 = m_1 = \frac{-2}{1} = -2$

Slope of $L_2 = m_2 = \frac{-2}{-1} = 2$

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2} = \frac{2 - -2}{1 + -2\times 2} = \frac{4}{-3}$$

$$\theta = \tan^{-1}\left(\frac{-4}{3}\right) = -53^0 7' 48'' (nearly)$$

But inverse t-function is not in present I year syllabus.

b) Equation is $L_1 + kL_2 = 0$

$$2x + y - 4 + k(2x - y - 2) = 0 \text{ ....}(1)$$

$$(2 + 2k)x + (1 - k)y - 4 - 2k = 0$$

$$Slope = \frac{-A}{B} = \frac{-(2 + 2k)}{1 - k} = \frac{2 + 2k}{k - 1}$$

Slope of the line $= \tan 45 = 1$

$$\Rightarrow \frac{2+2k}{k-1}=1$$
$$2+2k=k-1$$
$$2k-k=-1-2$$
$$k=-3$$

$in\ (1) \Rightarrow 2x+y-4+-3(2x-y-2)=0$

$2x+y-4-6x+3y+6=0$

$-4x+4y+2=0$

$2x-2y-1=0$

c) $2x-2y=1$

$$\frac{x}{\left(\frac{1}{2}\right)}+\frac{y}{\left(-\frac{1}{2}\right)}=1$$

Hence, x intercept $=\frac{1}{2}$

y intercept $=-\frac{1}{2}$

20. a) $c=4$

Equation in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

It passes through (3, 1)

$$\frac{3^2}{a^2}+\frac{1^2}{b^2}=1$$

$$\frac{a}{a^2}+\frac{1}{b^2}=1 \Rightarrow \frac{ab^2+a^2}{a^2b^2}=1 \Rightarrow a^2+ab^2=a^2b^2$$

Also $c=4$

$c^2 = 16$

$a^2 - b^2 = 16$

$a^2 = 16 + b^2 ....(2)$

$16 + b^2 + ab^2 = (16 + b^2)b^2$

$16 + 10b^2 = 16b^2 + b^4$

$b^4 + 16b^2 - 16 = 0$

$put\, b^2 = u$

$u^2 + 64 - 16 = 0$ $\quad || \quad Sum = 6; \quad product = -16$

$(4+8)(4-2) = 0$

$u = -8\ (or)\ u = 2$

$u = -8,\ \ is\, not\ possible.$

$\therefore u = 2 \Rightarrow b^2 = 2$

$a^2 - b^2 = c^2 \Rightarrow a^2 - 2 = 16$

$a^2 = 16 + 2 = 18 \Rightarrow a = \sqrt{18} = 3\sqrt{2}$

Length of major axis = 2a $= 2 \times 3\sqrt{2} = 6\sqrt{2}$

b) Standard equatin of the ellipse is $\frac{x^2}{18} + \frac{y^2}{2} = 1$

c) Eccentricity, $e = \frac{c}{a} = \frac{4}{3\sqrt{2}}$

Length of latus recturm $\frac{2b^2}{a} = \frac{2 \times 2}{3\sqrt{2}} = \frac{4}{3\sqrt{2}}$.

21. a) $\sin 75 = \sin(45 + 30)$

$= \sin 45.\cos 30 + \cos 45.\sin 30$

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

b) i) Length of arc $BDC = \frac{10\pi}{3}$

$r\theta = \frac{10\pi}{3}$

$\theta = \frac{10\pi}{3} \div r = \frac{10\pi}{3} \times \frac{1}{4} = \left(\frac{10\pi}{12} \times \frac{180}{\pi}\right)^0 = 150^0$

$\therefore \angle A = \frac{1}{2} \times 150^0 = 75^0$

ii) $\therefore \angle B = 180 - (75 + 60) = 45^0$

$$\frac{AB}{\sin 60} = \frac{4\sqrt{2}}{\sin 45}$$

$$AB = \frac{4\sqrt{2}}{\frac{1}{\sqrt{2}}} \times \frac{\sqrt{3}}{2}$$

$$= \frac{4\sqrt{2} \times \sqrt{2} \times \sqrt{3}}{2} = 4\sqrt{3} \text{ units}$$

$$\frac{BC}{\sin 75} = \frac{AB}{\sin 60}$$

$$BC = \frac{4\sqrt{3}}{\frac{\sqrt{3}}{2}} \times \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{4\sqrt{3} \times 2}{\sqrt{3}} \times \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$= \frac{8\left(\sqrt{3}+1\right)}{2\sqrt{2}} \text{units}$$

22 a) $\frac{3(x-2)}{5} \le \frac{5(2-x)}{3}$

$$9(x-2) \le 25(2-x)$$

$$9x - 18 \le 50 - 25x$$

$$9x + 25x \le 50 + 18$$

$$34x \le 68$$

$$x \le 2$$

$$\therefore x = (-\infty, 2]$$

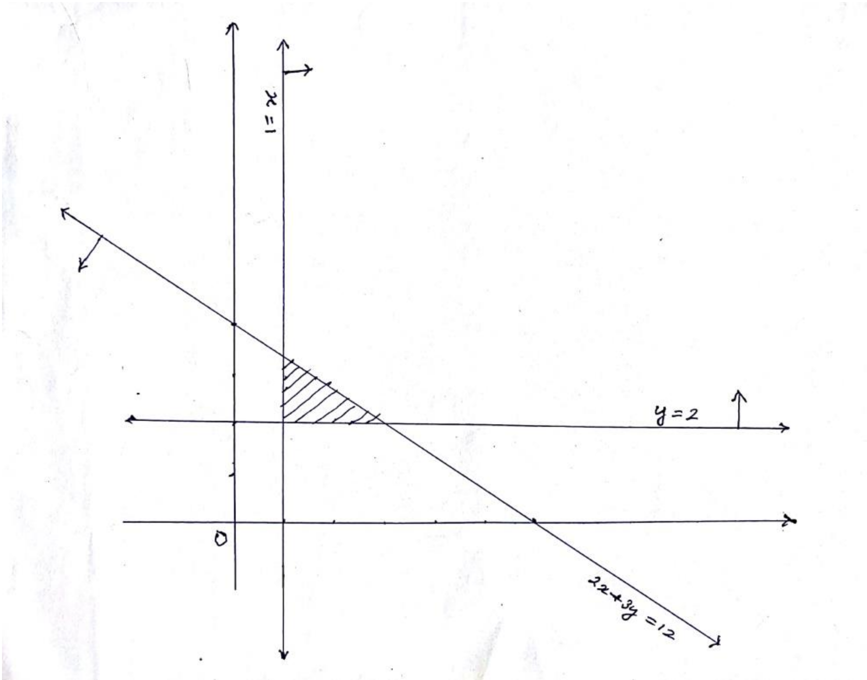
b)

$$2x + 3y = 12$$

x	0	6
y	4	0

$$x = 1$$

$$y = 2$$



23. a) $y = x^2 \Rightarrow f(x) = x^2$

$$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$$

$$f(x+h) - f(x) = (x+h)^2 - x^2$$

$$f'(x) = \lim_{x\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{x\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$= \lim_{x\to 0} \frac{2xh + h^2}{h} = \lim_{x\to 0} (2x + h) = 2x + 0 = 2x$$

b) $y = \dfrac{x}{1+\tan x}$

$$\frac{dy}{dx} = \frac{(1+\tan x)(1) - x(0 + \sec^2 x)}{(1+\tan x)^2} = \frac{1 + \tan x - x\sec^2 x}{(1+\tan x)^2}$$

24. a) $n = \dfrac{99-3}{3} + 1 = \dfrac{96}{3} + 1 = 32 + 1 = 33$

b) $\sum x_i = 3 + 6 + 9 + ... + 99$

$$= \frac{n}{2}(a + a_n) = \frac{33}{2}(3 + 99)$$

$$= \frac{33}{2} \times 102 = 33 \times 51 = 1683$$

$$Mean,\ \bar{x} = \frac{\sum x_i}{n} = \frac{1683}{33} = 51$$

c) $\sum x^2 = 3^2 + 6^2 + 9^2 + ... + 99^2$

$$= 3^2\left(1^2 + 2^2 + 3^2 + ... + 33^2\right)$$

$$= \frac{9 \times 33(33+1)(2 \times 33+1)}{6} \qquad \| S_n = \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{9 \times 33 \times 34 \times 67}{62} = 9 \times 11 \times 17 \times 67 = 112761$$

d) Variance $= \frac{\Sigma x_i^{\ 2}}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$$= \frac{112761}{33} - 51^2$$

$$= 3417 - 2601 = 861$$

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